Abstract

Omnipresence of heterogeneity is conspicuous in all creations of nature. Heterogeneity manifests itself in many forms at different scales, both in time and space. Engineering domain being an exotic fusion of human creativity and ever-increasing demands exemplifies the ubiquity of heterogeneity. Surprisingly, the plethora of materials we see around seem to stem from myriad combination of few *base* materials identified as *elements* in chemistry. Further, a simple rearrangement of atoms in these materials leads to allotropes with startling contrasts in properties. Similarly, micro- and meso-scales in heterogeneous materials also display this phenomenon. Human requirements propelled by necessities and wants have leveraged heterogenity deliberately or naively. In the context of engineering materials, light weight heterogeneous materials like composites and cellular solids are outstanding inventions from the last century.

The present thesis highlights this phenomenon on a meso-scale to explore *generalized* variants of *circular* and *elliptical* honeycomb structures (HCSs) with an emphasis on their *effective transverse elastic responses*, a crucial pillar of engineering design and analysis. Homogenized or effective properties are an extension of continuum hypothesis, conceived for ease in analyses. Effective properties are employed in multi-scale analyses resulting in less complex models for analysis, for example, for predicting the speed of wave propogation.

The thesis extends and generalizes existing *close-packed* circular and elliptical HCSs to more broader configurations. Simpler periodic arrangement of the unit cells from numerous exotic possibilities directly incorporates Design for Manufacture and Assembly (DFMA) philosophy and offers a potential scope for analysis by simpler tools resulting in handy expressions which are of great utility for designer engineers. In this regard, analytical expressions for moduli having compact forms in the case of circular HCS are developed by technical theories and rigorous theory of elasticity. Regression analysis expressions for the moduli of elliptical HCS are presented, and the elasticity solutions for the same are highlighted.

The thesis consists of seven chapters with **Chapter 1** presenting generalized circular and elliptical HCSs as a potential avenue beyond composite materials. Following a survey of pertinent HCS literature of these HCSs, research gaps and

scope are delineated.

Chapter 2 briefly summarizes the ideas, concepts and tools including analytical and numerical methods. This chapter sets the ground for the analysis of generalized circular and elliptical HCS in the following four chapters.

Following the classification of the circular HCSs, **Chapter 3** assesses the complete transverse elastic responses of generalized circular HCS through *technical theories* which are a first-order approximation. Here, thin ring theory and the more elaborate curved beam theory are employed as models to assess the moduli. Normal moduli - E^* and ν^* - are obtained by employing Castigliano method, while shear moduli (G^*) are obtained by solving the differential equations derived in terms of displacements. Compact expressions for moduli presented wherever possible furnish the designer with a range of moduli for different configurations and modular ratios (E_y/E_x). The results show the range of applicability of technical theories within 5% of FEA. For hexagonal arrays, these results are more *refined* than those in literature; while the same are new for other configurations. Surprisingly, the more elaborate curved beam theory offers no better results than the thin ring theory.

Chapter 4 extends the aforementioned task of assessing the complete transverse elastic moduli of generalized circular HCS by employing rigorous theory of elasticity (TOE) which is a second-order approximation. Utilizing Airy stress function in polar coordinates, the boundary value problems resulting from modeling of the circular HCS under different loads are solved analytically in conjunction with FEA employing *contact* elements. Contact elements circumvent the point loads which give finite values of displacements in technical theories and singular values in TOE. A widely used idea of employing distributed load, statically equivalent to point load, is invoked to empower TOE. The distributed load is assumed *a priori* and the contact length is obtained from FEA employing contact elements. Thus, FEA compliments the present analytical methods. Results demonstrate a very good match between analytical method in conjunction with FEA and numerical results from FEA; the error is within 5% for very thick ring (thickness-radius ratio ≈ 0.5). Further, computationally and numerically efficient expressions for displacements give better results with same computational facility.

To illustrate the effect of coating on effective moduli, a limited study based on thin ring theory and elasticity theories is undertaken in **Chapter 4**. The study explores the effects of moduli and thickness ratios of substrate to coating on the effective normal moduli. Employing thin ring theory with only flexure as the bending mode, we get compact expressions giving good match for very thin rings in all configurations. The elasticity approach presented for square array demonstrates a very good match with FEA for thick rings. Coatings offer a strategy to increase the effective moduli with same dimensions.

Chapter 5 broadens the scope of circular HCS by considering *elliptical* HCSs. While generalized circular HCS can cater to anisotropic requirement to an extent, larger spectrum is offered by considering *elliptical* honeycomb structures. In this regard, a generalized version of *concentric* thin coated elliptical HCS is investigated for transverse moduli. Thin HCSs are explored by technical theories as in circular HCS. However, a lack of exact compact-form expressions necessitates the use of regression analysis. The resulting expressions are presented in terms of ellipticity ratio describing the ovality of the ellipse and geometric parameters. Normal moduli are obtained by Castigliano method implemented in MATHEMATICA, but shear moduli are obtained from FEA employing *beam* elements. The need for FEA employing beam elements stems from the subtle fact that Castigliano method implicitly assumes preclusion of rigid body motions, while shear loading for shear moduli evaluation entails rigid body motions. Interestingly, curved beam theory, as in circular HCS, offers no better refinement in assessing the moduli as compared to thin ring theory. The graphs showing the moduli with respect to thickness and modular ratios are presented as design maps to aid the designer.

Chapter 6 extends the works of thin *concentric* coated elliptical to *thicker concentric* and a *novel confocal* elliptical HCS, a variant of elliptical HCS. In this regard, thick concentric and confocal elliptical HCS by elasticity approach are attempted for a simple case. Airy stress function in polar coordinates is tried for concentric elliptical HCS. Confocal HCS analysis employs stress function in terms of elliptical coordinate system. After proving the correctness of the stress function for both the cases by comparing the reconstructed boundary conditions with actual boundary conditions, the restrictions in solving the case of rings under load over a *small region* is highlighted. A parametric study for moduli is undertaken by employing FEA. These are presented as design graphs which compare and contrast the two variants of elliptical HCS on the same graphs. The modular ratio (E_y/E_x) is conspicuously *more* for confocal elliptical HCS than concentric elliptical HCS.

Chapter 7 gives the conclusions in a nutshell, and explores the feasibility of stress evaluation of heterogeneous media on the lines of effective media theory.